[13.32] Show that every finite group **G** has a faithful representation in GL(*n*) where *n* is the order of **G.**

**Solution**

A representation is a function  that preserves the group structure; i.e., for all  *T* is faithful if it is one-to-one; i.e., if 

**Part A**. Show T is a representation

I do this 2 ways. I do it my way first, then I repeat it using Robin’s method which is very slick. For motivation, I use Penrose’s hint to label the matrix for *T*(*g i*).



Given *k* there is a unique *j* such that  So I think of this matrix as  (Robin had which also works, as does right multiplication by *gi*.) Thus I make the definition



which in matrix notation is 

It suffices to show that  on each basis element 

From definition (1) we get that

 or 

and



Thus,



Again, from definition (1), we get that

 or

 So 

So, 

That is,  ✔

**Alternate proof of Part A**



 Therefore  But,

 ✔

**Part B**  Show *T* is faithful

We suppose  Since 

 

 and  So for all *a* and *b* we have  Letting *b* = *e* (the identity of **G**) we get

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